Martin Ziegler
Holger Thies

Issued on 2015-04-21

Computational Complexity in Analysis

SoSe 2015, Exercise Sheet #2

EXERCISE 3:

Consider the set $\mathcal{NP}_1$ of languages of the form

$$\big\{ \mathtt{1}^n \,:\, \exists \vec{w} \in \{\mathtt{0},\mathtt{1}\}^{p(n)} : (\mathtt{1}^n\,\mathtt{0}\,\vec{w}) \in V \big\}, \qquad V \in \mathcal{P}, \quad p \in \mathbb{N}[N] \ .$$

a) Prove $\mathcal{P}_1 = \mathcal{NP}_1 \Rightarrow \mathsf{EXP} = \mathsf{NEXP}$.

b) How about the converse?

*) Can $\mathcal{NP}_1$ contain $\mathcal{NP}$-complete languages? Prove or disprove!

EXERCISE 4:

a) Prove that sum, product, and reciproke of polynomial-time computable real numbers are again polynomial-time computable.

b) Prove that every algebraic real is computable in polynomial time.
More generally (why?) every real root of a polynomial with coefficients computable in polynomial time is again polynomial-time computable.

c) Prove that transcendental numbers e and π are polynomial-time computable.