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# CS422 Dec.17, 2015 <u>Final Exam</u>

Please write your name and student ID here ______________________________

as well as on each additional sheet of paper you use!

30 points = 100%

**Problem 1 (10 points):** Please classify the following decision problems according to their computational complexity by marking one box in each of the following rows:

▣ $\mathcal{P}$

▣ $\mathcal{NP}$−complete

▣ not (known to be) in $\mathcal{NP}$

☐ ☐ ▣ a) Given an algorithm (**`WHILE+`** program) $\mathcal{A}$ with quadratic runtime; does there exist an input it accepts?

▣ ☐ ☐ b) Given a finite <u>automaton</u> $\mathcal{A}$; does there exist an input $\underline{x}$ it accepts?

☐ ☐ ▣ c) Given a multivariate integer polynomial $p=p(Y_1,\ldots,Y_n)\in\mathbb{Z}[Y_1,\ldots,Y_n]$; does it have an <u>integer</u> root, i.e., $(y_1,..,y_n)\in\mathbb{N}^n$ s.t. $p(y_1,..,y_n)=0$ ?

☐ ☐ ▣ d) Given a multivariate integer polynomial; does it have a <u>real</u> root?

▣ ☐ ☐ e) Given a finite string of brackets **(** and **)**, are they correctly nested?

☐ ▣ ☐ f) Given two Boolean expressions $\Phi$ and $\Psi$; are they *non*−equivalent?

☐ ☐ ▣ g) Given a Boolean expression $\Phi$; does there exist a shorter, equivalent one?

☐ ☐ ▣ h) Given a Boolean expression $\Phi=\Phi(Y_1\ldots Y_n)$, does it hold $\exists y_1\in\{0,1\}\ \forall y_2\in\{0,1\}\ \exists y_3\in\{0,1\}\ \forall y_4 \ldots \exists/\forall y_n\in\{0,1\}: \Phi(y_1,\ldots,y_n)=1$

▣ ☐ ☐ i) Given a natural number $N$, is it composite (i.e. *non*−prime) ?

▣ ☐ ☐ j) Given a graph $G$; can it be drawn on the plane without crossings?

No justification or proofs are required here!

**Problem 2 (5+5 points): a)** Devise* a (direct and explicit) polynomial-time reduction from Boolean satisfiability in 5-CNF (conjunction of disjunctions of five literals each)

**5-SAT**= $\{ \langle\Phi(Y_1\ldots Y_n)\rangle$ : Boolean term $\Phi$ in 5-CNF admits a satisfying assignment $y_1\ldots y_n$ $\}$

to Integer Linear Program **ILP** = $\{ \langle \underline{A},\underline{b}\rangle : \underline{A}\in\mathbb{Z}^{m\times n}, \underline{b}\in\mathbb{Z}^m, \exists \underline{x}\in\mathbb{N}^n: \underline{A}\cdot\underline{x}=\underline{b} \}$.

**b)** Devise* a polynomial-time reduction from the Hamilton Circuit Problem

**HC** = $\{ \langle G\rangle$ : graph $G=(V,E)$ admits a complete cycle visiting each vertex precisely once $\}$

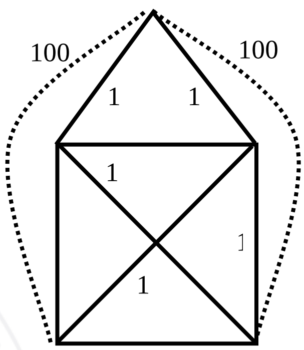
to the Travelling Salesperson Problem of whether a given complete graph with edge weights $\underline{c}$ admits a complete cycle of weight at most $k$:

**TSP** = $\{ \langle \underline{c},k\rangle \mid \underline{c}:\{0,\ldots n\text{-}1\}\times\{0,\ldots n\text{-}1\}\to\mathbb{N}$,

$\exists$ bijection $\pi:\{0,\ldots n\text{-}1\}\to\{0,\ldots n\text{-}1\}$:

$k \geq \sum_{0\leq j<n} c\big(\pi(j),\pi(j+1 \bmod n)\big) \}$

Hint: Assign edges $e$ absent in $G$ to have 'large' weight $\underline{c}(e)$. How large?




**Problem 3 (1+2+3 points):** The lecture established the following problem as $\mathcal{NP}$-complete:

**UNP** = $\{ \langle \mathcal{A},x,2^N\rangle$ : nondetermin. **WHILE+** program $\mathcal{A}$ accepts input $x$ within at most $N$ steps $\}$

**a)** Define a similar $\mathcal{PSPACE}$-complete problem **UPSPACE**,

**b)** show **UPSPACE** to belong to $\mathcal{PSPACE}$

**c)** and reduce every problem $L\in\mathcal{PSPACE}$ to **UPSPACE** in polynomial time.

Recall that the universal **`WHILE+`** program $\mathcal{U}$ can simulate a given $\mathcal{A}$ on given input $x$ using memory $O\big(\ell(\langle\mathcal{A}\rangle)+\ell(x)\big)$ in addition to what $\mathcal{A}$ itself uses on $x$.

**Problem 4 (4 points):** Recall that **SAT** = $\{ \langle\Phi(Y_1\ldots Y_n)\rangle : \Phi$ Boolean term, $\exists y_1\ldots y_n : \Phi(y_1,\ldots,y_n)=1 \}$ belongs to $\mathcal{NP}$ but its complement **SAT**$^C$ = $\{ \langle\Phi(Y_1\ldots Y_n)\rangle : \forall y_1\ldots y_n : \Phi(y_1,\ldots,y_n)=0 \}$ does probably not, nor does $\{ \langle\Phi(Y_1\ldots Y_n)\rangle \mid \exists\Psi: \ell(\langle\Psi\rangle)<\ell(\langle\Phi\rangle), \forall y_1\ldots y_n : \Phi(y_1\ldots y_n)=\Psi(y_1\ldots y_n) \}$.

Now prove that the following problem belongs to $\mathcal{PSPACE}$:

**QBF** = $\{ \langle\Phi(Y_1\ldots Y_n)\rangle : \Phi$ Boolean term, $\exists y_1\, \forall y_2\, \exists y_3\, \forall y_4 \ldots \exists/\forall y_n: \Phi(y_1,\ldots,y_n)=1 \}$.

**Problem 5 (0 points):** Which textbook (title, author/s) did you buy to accompany this lecture?

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* Describe a translation function, analyze the runtime of your algorithm computing it, and prove the reduction property.