Martin Ziegler
MiJin Kim

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CS422

Fall 2015, Assignment #2

SOLUTION 4:

b) $L := \{\langle\mathcal{A}\rangle : \exists\vec{x} : \mathcal{A} \text{ does not terminate on } \vec{x}\}$.
The hypothesis that some algorithm $\mathcal{B}$ semi-decides L implies

$$\mathcal{B} \text{ on } \langle\mathcal{B}\rangle \text{ terminates} \quad \Leftrightarrow \quad \exists\vec{x}: \ \mathcal{B} \text{ on } \vec{x} \text{ does not terminate}$$

which by itself does *not* lead to a contradiction!

SOLUTION 5:

a) The mapping $\langle\mathcal{A}\rangle \mapsto \langle\mathcal{A}, \texttt{"stop"}\rangle$ is a computable reduction.

b) The mapping $\langle\mathcal{A}\rangle \mapsto \langle\mathcal{A}'\rangle$ is a computable reduction, where we design $\mathcal{A}'$ such that

- on the empty input it stops right away;
- on inputs $\texttt{0}\vec{x}$ and $\texttt{1}\vec{x}$ it simulates $\mathcal{A}$ on input $\vec{x}$:

For $\langle\mathcal{A}\rangle \in T$, $\mathcal{A}'$ terminates on both the empty and every non-empty input, hence $\langle\mathcal{A}'\rangle \in X$.
For $\langle\mathcal{A}\rangle \notin T$, there exists some input $\vec{x}$ which $\mathcal{A}$ does not terminate on;
hence $\mathcal{A}'$ terminates on the empty input but not on $\texttt{0}\vec{x}$; thus showing $\langle\mathcal{A}'\rangle \notin X$.

c) The mapping $\langle\mathcal{A}\rangle \mapsto \langle\mathcal{A}''\rangle$ is a computable reduction, where we design $\mathcal{A}''$ such that

i) on input $\langle\vec{x},\vec{y},N\rangle$
ii) simulate $\mathcal{A}$ on input $\vec{x}$ for N steps:
iii) stop if $\mathcal{A}$ on input $\vec{x}$ does *not* terminate within N steps;
iv) otherwise proceed to simulate $\mathcal{A}$ on input $\vec{y}$ for indefinitely many steps.

- If $\mathcal{A}$ terminates for every input $\vec{y}$ (and hence $\langle\mathcal{A}\rangle \in X$),
  then $\mathcal{A}''$ terminates for every input $\langle\vec{x},\vec{y},N\rangle$ either in (iii) or in (iv).
- If $\mathcal{A}$ terminates for no input $\vec{x}$ (and hence $\langle\mathcal{A}\rangle \in X$),
  then $\mathcal{A}''$ terminates for every input $\langle\vec{x},\vec{y},N\rangle$ in (iii).
- If $\langle\mathcal{A}\rangle \notin X$, then there exists some input $\vec{x}$ on which it does terminate (say, after N steps) and some input $\vec{y}$ on which it does not terminate.
  Then $\langle\vec{x},\vec{y},N\rangle$ constitutes an input which $\mathcal{A}''$ does not terminate on: $\langle\mathcal{A}''\rangle \notin T$.