KAIST School of Computing
291 Daehak-ro, Yuseong-gu, Daejeon 305-701, Republic of Korea
Tel. +82-42-350-3502~3507 Fax. +82-42-350-3510

Prof. Dr. Martin Ziegler
+82-42-350-3568
ziegler@kaist.ac.kr
http://m.zie.de

CS700 Dec.15, 2016 <u>Final Exam</u>

Please write your name and student ID here ______________________________

as well as on each additional sheet of paper you use!

30 points = 75%

Problem 1 (3+2+2+3 points):

a) Give an example of a continuous but *in*computable function $f:[0;1]\to\mathbb{R}$.

b) Specify a *decidable* (discrete) L that can*not* be decided in *polynomial* time unless $\mathcal{P}=\mathcal{NP}$.

c) Give an example of an *computable* function $h:[0;1]\to\mathbb{R}$
that can*not* be computed in *polynomial* time.

d) Justify your answer in c).

Recall that a modulus of continuity $\mu:\mathbb{N}\to\mathbb{N}$ of $f:[0;1]\to\mathbb{R}$ by definition satisfies
$|x-x'|\leq 2^{-\mu(n)} \Rightarrow |f(x)-f(x')|\leq 2^{-n}$ and L-Lipschitz means $|f(x)-f(x')|\leq L\cdot|x-x'|$

Problem 2 (3+2+2+3 points):

a) Let $f:[0;1]\to[0;1]$ have modulus of continuity μ and $g:[0;1]\to[0;1]$ have modulus ν.
Prove that their composition $g\circ f$ has modulus $\mu\circ\nu$.

b) For L-Lipschitz $f:[0;1]\to[0;1]$ and K-Lipschitz $g:[0;1]\to[0;1]$, $g\circ f$ is $L\cdot K$-Lipschitz.

c) For $L=2=K$ give an example showing that b) is optimal!

d) Prove that the function $h:[0;1]\ni x\to 1/\ln(e/x)\in[0;1]$ is well-defined and continuous
but does not admit a *polynomial* modulus of continuity.

Problem 3 (5+5 points):

a) Prove (without recurring to results from the lecture) that there exist (not necessarily computable) sequences $(a_n),(b_n)\subseteq\mathbb{Q}$ such that $\mathbb{R}_c\subseteq\bigcup_n (a_n,b_n)$ and $\sum_n |b_n-a_n|\leq ½$.

b) Describe, establish correctness, and analyze the runtime of an algorithm computing the *minimum* of an arbitrary but fixed polynomial-time computable 1-Lipschitz $f:[0;1]\to[0;1]$.

Problem 4 (0 points): Remember that

a) there is a computable increasing bounded rational sequence with *in*computable limit;

b) there is a computable <u>smooth</u> $f:[0;1]\to\mathbb{R}$ attaining its minimum in *no* computable point;

c) there is a computable continuously differentiable $f:[0;1]\to\mathbb{R}$ with *in*computable derivative.

d) Every real function computable in time $t(n)$ has modulus of continuity $t(n+1)+1$;

e) every computable $f:[0;1]\to\mathbb{R}$ is computable in some time bound $t(n)$ depending only on the output precision n; and has computable maximum and computable integral.